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REVIEWS

Fluid Mechanics and Singular Perturbations: A Collection of Papers by Saul Kaplun. Edited by P. Lagerstrom, L. N. Howard and C.-S. Liu. Academic Press, 1967. 369 pp. $10.00 or £4.

Our understanding and command of singular-perturbation theory in fluid mechanics has been significantly advanced in the past two decades by the work of Professor Lagerstrom and his students, notably Julian Cole, Milton Van Dyke, and the late Saul Kaplun. This book collects Kaplun's three published papers and much of his unpublished work in edited form.

Part one, edited by Lagerstrom and Howard, comprises the published (chapters I–III) and two unpublished papers (chapters IV and V) on the asymptotic theory of the Navier–Stokes equations and, in addition, an essentially mathematical paper (chapter VI) on the structure of partially ordered spaces. Kaplun's important contributions in the published papers, including the systematic use of inner and outer limits and the clarification of the important role of the co-ordinate system in the method of matched asymptotic expansions, are now well known. Chapter IV, parts of which are anticipated in the published papers, gives a penetrating discussion of inner and outer limits and of the crucial question of matching; it is perhaps the most important chapter in the book (granted the previous availability of chapters I–III). Chapter V deals briefly with the two-dimensional lifting problem at low Reynolds numbers, using singular-perturbation methods. Chapter VI is likely to be of greater interest for mathematics than fluid mechanics; in any event, I found it difficult reading and am not competent to assess its importance.

Part two, edited by Lagerstron and Liu, comprises Kaplun's unpublished work on boundary-layer separation and deals principally with the nature of the solution to Prandtl's boundary-layer equations in the neighbourhood of zero skin friction. This problem has been studied by Kármán & Millikan (1934), Howarth (1938), Hartree (1939), Goldstein (1948), Jones (1948), Leigh (1955), Stewartson (1958), Landau & Lifshitz (1959), and Terrill (1969) [see L. Rosenhead, *Laminar Boundary Layers* (Oxford University Press, 1963), pp. 217–21 for discussion and references], but Kaplun refers only to the work of Hartree and Goldstein. The starting point for Kaplun's work is Goldstein's remark that the solution to Prandtl's boundary-layer equations contains an algebraic singularity at the point of separation. Goldstein's solution, based on a co-ordinate expansion, arbitrarily requires a certain integral, say I, to vanish and does not contain a sufficient number of arbitrary constants to permit a proper matching. Stewartson's work reveals that I need not vanish and that additional, logarithmic terms must be added to Goldstein's expansion. Kaplun, who does not appear to have read Stewartson's paper, uses a parameter expansion and obtains results that are at least partially equivalent to those of Stewartson; however, the editors do not attempt a comparison of Kaplun's work with that of Stewartson 'since Kaplun arrived at his basic results quite independently, and none of his manuscripts contain such a comparison'. This is, I think, unfortunate, especially in view of the unresolved controversy over the necessity of a singularity at the point of separation. The editors refer to this controversy only through the rather oblique remark that 'Some authors have assumed that the outer flow and the boundary layer (*sic*) interact in such a way that the pressure distribution along the body takes on the highly special form needed to avoid [a singularity at the point of separation].† Kaplun dismissed this possibility and in fact assumed that a constant pressure gradient would be representative'. 'Some authors' presumably refers to Stewartson, who states that such a singularity is not necessary (Stewartson's position appears to be supported by C. François, *La Recherche Aérospatiale*, no. 121, 1967, 17–28). Landau & Lifshitz (1959, p. 152), reporting earlier work by Landau along somewhat different lines, appear to support Kaplun's position with the conclusion that 'separation can occur only on a line whose points are singularities of the solution of Prandtl's equations'; however, neither Kaplun nor his editors appear to have been aware of Landau's work.

The editors clearly intended this book as a memorial to a colleague whose premature death deprived us of much that would have been important for both fluid mechanics and applied mathematics. It is not possible to criticize their motives, but I cannot avoid the surmise that they may have done Kaplun a disservice in publishing his work on separation without a more adequate attempt to fit it into the existing literature. Such an attempt is made (successfully in my opinion) in the Introduction to Part I, which places Kaplun's work in clear, historical perspective and underscores the capital importance of his contributions to singular-perturbation theory. The editors' failure to provide this perspective for Part II leaves us with no more than a historical record of work that does not appear to fit directly into the mainstream of scientific literature. Still, the record is there, and with this we must rest content.

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† Professor Lagerstrom informs me that the misprinted word 'spearation' on p. 149 should be replaced by the bracketed phrase.